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EP2.

Problema 4- Decomposição LU

$$A = \begin{bmatrix} 18 & -9 & -3 \\ 3 & 15 & -9 \\ 1 & 1 & -3 \end{bmatrix} = L \cdot U \quad ; \quad b = \begin{bmatrix} 13 \\ 8 \\ 4 \end{bmatrix}$$

Precisamos determinar arbitrariamente valores para os termos da diagonal principal da matriz L

Vamos escolher então $l_{11} = l_{22} = l_{33} = 1$

Temos então:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad e \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Encontrar L e U pelo algoritmo de Crout

<u>Algoritmo</u> <u>de</u> <u>Crout</u>:

Para $j = 1, \dots, 3$:

a-) Para $i = 1, \dots, j$:

$$u_{ij} = a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj}$$

b-) Para $i = j+1, \dots, 3$:

$$l_{ij} = \frac{1}{u_{jj}} \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \right)$$

$j = 1:$

$i = 1:$

$$u_{11} = a_{11} - \sum_{k=1}^{0} l_{1k} u_{k1} = a_{11} \Rightarrow u_{11} = 18$$

$i = 2:$

$$l_{21} = \frac{1}{u_{11}} \cdot \left(a_{21} - \sum_{k=1}^{0} l_{2k} u_{k1} \right) = \frac{a_{21}}{u_{11}} \Rightarrow l_{21} = \frac{3}{18} = \frac{1}{6}$$

$i = 3:$

$$l_{31} = \frac{1}{u_{11}} \cdot \left(a_{31} - \sum_{k=1}^{0} l_{3k} u_{k1} \right) = \frac{a_{31}}{u_{11}} \Rightarrow l_{31} = \frac{1}{18}$$

$j = 2:$

$i = 1:$

$$u_{12} = a_{12} - \sum_{k=1}^{0} l_{1k} u_{k2} = a_{12} \Rightarrow u_{12} = -9$$

$i = 2:$

$$u_{22} = a_{22} - \sum_{k=1}^{1} l_{2k} u_{k2} = a_{22} - l_{21} u_{12} \Rightarrow u_{22} = 15 - \frac{1}{6} \cdot (-9)$$

$$\Rightarrow u_{22} = 15 + \frac{9}{6} = 15 + \frac{3}{2} \Rightarrow u_{22} = \frac{33}{2}$$

$i = 3:$

$$l_{32} = \frac{1}{u_{22}} \left(a_{32} - \sum_{k=1}^{1} l_{3k} u_{k2} \right) = \frac{1}{u_{22}} \left(a_{32} - l_{31} u_{12} \right)$$

$$\Rightarrow l_{32} = \frac{1}{\left(\frac{33}{2}\right)} \left(1 - \frac{1}{18} (-9) \right) = \frac{2}{33} \left(1 + \frac{1}{2} \right) = \frac{2}{33} \cdot \frac{3}{2}$$

$$\Rightarrow l_{32} = \frac{1}{11}$$

$j = 3:$

$i = 1:$

$$u_{13} = a_{13} - \sum_{k=1}^{0} l_{1k} u_{k3} = a_{13} \Rightarrow u_{13} = -3$$

$i = 2$:

$$u_{23} = a_{23} - \sum_{k=1}^{1} l_{2k} u_{k3} = a_{23} - l_{21} u_{13}$$

$$\Rightarrow u_{23} = -9 - \frac{1}{6} \cdot (-3) = -9 + \frac{1}{2} \Rightarrow u_{23} = -17/2$$

$i = 3$:

$$u_{33} = a_{33} - \sum_{k=1}^{2} l_{3k} u_{k3} = a_{33} - \left(l_{31} u_{13} + l_{32} u_{23} \right)$$

$$\Rightarrow u_{33} = -3 - \left(\frac{1}{18} (-3) + \frac{1}{11} \cdot \left(- \frac{17}{2} \right) \right)$$

$$\Rightarrow u_{33} = -3 + \left(\frac{1}{6} + \frac{17}{22} \right) = \frac{-396 + 22 + 102}{132}$$

$$\Rightarrow u_{33} = \frac{-272}{132} \Rightarrow u_{33} = \frac{-68}{33}$$

Temos então:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/6 & 1 & 0 \\ 1/18 & 1/11 & 0 \end{bmatrix} \quad e \quad U = \begin{bmatrix} 18 & -9 & -3 \\ 0 & 33/2 & -17/2 \\ 0 & 0 & -68/33 \end{bmatrix}$$

$$A = LU \Rightarrow Ax = LUx = L(Ux) = Ly, \; y = Ux$$

Vamos primeiro encontrar y tal que $Ly = b$ e depois encontrar x tal que $Ux = y$

$$Ly = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/6 & 1 & 0 \\ 1/18 & 1/11 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 8 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y_1 + 0y_2 + 0y_3 = 13 & (1) \\ 1/6\, y_1 + y_2 + 0y_3 = 8 & (2) \\ 1/18\, y_1 + 1/11\, y_2 + y_3 = 4 & (3) \end{cases}$$

$y_1 = 13 \Rightarrow$ equação (1)

Substituindo y_1 em (2), temos

$$\frac{13}{6} + y_2 = 8 \Rightarrow y_2 = 8 - \frac{13}{6} = \frac{48-13}{6} \Rightarrow y_2 = \frac{35}{6}$$

Substituindo y_1 e y_2 em (3), temos

$$\frac{13}{18} + \frac{35}{6} \cdot \frac{1}{11} + y_3 = 4 \Rightarrow \frac{13}{18} + \frac{35}{66} + y_3 = 4$$

$$\Rightarrow y_3 = 4 - \frac{13}{18} - \frac{35}{66} = \frac{4752 - 858 - 630}{1188} = \frac{3264}{1188} \Rightarrow y_3 = \frac{272}{99}$$

$$U_x = y \Rightarrow \begin{bmatrix} 18 & -9 & -3 \\ 0 & 33/2 & -17/2 \\ 0 & 0 & -\frac{68}{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 35/6 \\ 272/99 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 18x_1 - 9x_2 - 3x_3 = 13 & (4) \\ 0x_1 + \frac{33}{2}x_2 - \frac{17}{2}x_3 = \frac{35}{6} & (5) \\ 0x_1 + 0x_2 - \frac{68}{33}x_3 = \frac{272}{99} & (6) \end{cases}$$

Da equação (6), vemos que $x_3 = \dfrac{\left(\frac{272}{99}\right)}{\left(\frac{-68}{33}\right)} = \dfrac{-33 \cdot 272}{68 \cdot 99}$

$$\Rightarrow x_3 = \frac{-4}{3}$$

Substituindo na equação (5),

$$\frac{33}{2}x_2 + \frac{17}{2}\left(\frac{4}{3}\right) = \frac{35}{6}$$

$$\Rightarrow \frac{33}{2}x_2 = \frac{35}{6} - \frac{68}{6} \Rightarrow \frac{33}{2}x_2 = \frac{-33}{6} \Rightarrow x_2 = \frac{-33}{6}\cdot\frac{2}{33} \Rightarrow x_2 = \frac{-1}{3}$$

Substituindo x_2 e x_3 na equação (3),

$$18x_1 - 9\left(-\frac{1}{3}\right) - 3\cdot\left(-\frac{4}{3}\right) = 13$$

$$\Rightarrow 18x_1 + 3 + 4 = 13 \Rightarrow 18x_1 = 13 - 4 - 3 \Rightarrow 18x_1 = 6$$

$$\Rightarrow x_1 = \frac{6}{18} \Rightarrow x_1 = \frac{1}{3}$$

Encontramos então nosso vetor x:

$$x = \begin{bmatrix} 1/3 \\ -1/3 \\ -4/3 \end{bmatrix}$$